

Operations with sets – Union, Intersection and Complement

A **universal set** for a particular problem is a set which contains all the elements of all the sets in the problem.

A universal set is often denoted by a capital U, but sometimes the Greek letter ξ (see) is used.

In this section we will create subsets of a given universal set and use set operations to create new subsets of the universal set.

There are three set operations we will learn in this section.

- **Complement:** The complement of a set A is symbolized by A' and it is the set of all elements in the universal set that are not in A.
- **Intersection:** The intersection of sets A and B is symbolized by $A \cap B$ and is the set containing all of the elements that are common to both set A and set B.
- **Union:** The union of set A and B is symbolized $A \cup B$ and is the set containing all the elements that are elements of set A or of set B or that are in both Sets A and B.

Here is a quick example to illustrate the 3 definitions.

Example: Let U be a universal set and A and B be subsets of U defined as follows.

$$U = \{1,2,3,4,5\}$$

$$A = \{1,2,3\}$$

$$B = \{2,3,4\}$$

Find A'

A' is all of the elements in the Universal set that are not in set A.

$$\text{Answer: } A' = \{4,5\}$$

Find $A \cap B$ (This is asking me to find all of the elements that A and B have in common.)

$$\text{Answer: } A \cap B = \{2,3\}$$

Find $A \cup B$ (This is asking me to list all of the elements in A followed by all of the elements in B, then delete any elements that are written twice.)

$$A \cup B = \{1,2,3,2,3,4\}$$

$$\text{Answer: } A \cup B = \{1,2,3,4\}$$

Example: Let U be a universal set and A and B be subsets of U defined as follows.

$$U = \{a, b, c, d, e, f\}$$

$$A = \{a, b, c\}$$

$$B = \{c, d, e\}$$

Find $A' \cap B$

First I need to find A' , which is all of the elements in U that aren't in set A .

$$A' = \{d, e, f\}$$

Now I can intersect the two sets.

$$A' \cap B = \{d, e, f\} \cap \{c, d, e\} \text{ (now find what the two sets have in common)}$$

$$\text{Answer: } \{d, e\}$$

Find $A \cup B'$

First I need to find B'

$$B' = \{a, b, f\}$$

$$A \cup B' = \{a, b, c\} \cup \{a, b, f\} \text{ (put all 6 elements in a big set then delete the duplicates)}$$

$$= \{a, b, c, a, b, f\}$$

$$\text{Answer: } \{a, b, c, f\}$$

#1-10: Find the following sets.

$$U = \{a, b, c, d, e\} \quad A = \{c, d, e\} \quad B = \{a, c, d\}$$

$$1) A'$$

$$2) B'$$

$$3) A \cup B$$

$$4) A' \cup B'$$

$$5) A \cap B$$

$$6) A' \cap B'$$

$$7) A' \cap B$$

$$8) A \cap B'$$

$$9) A' \cup B$$

$$10) A \cup B'$$

#11-20: Find the following sets.

$$U = \{1,2,3,4,5\} \quad A = \{1,2,3\} \quad B = \{5\}$$

11) A'

12) B'

13) $A \cup B$

14) $A' \cup B'$

15) $A \cap B$

16) $A' \cap B'$

17) $A' \cap B$

18) $A \cap B'$

19) $A' \cup B$

20) $A \cup B'$

Example: Let U be a universal set and A , B and C be subsets of U defined as follows.

$$U = \{a,b,c,d,e,f\}$$

$$A = \{a,b,c\}$$

$$B = \{c,d,e\}$$

$$C = \{d,e,f\}$$

Find $A \cup B \cup C$

I need to work from left to right. First I will find $A \cup B$

$$A \cup B = \{a,b,c\} \cup \{c,d,e\}$$

$$= \{a,b,c,c,d,e\}$$

$$= \{a,b,c,d,e\}$$

Now I can do the union C part. I can rewrite my problem as:

$$\{a,b,c,d,e\} \cup C$$

$$= \{a,b,c,d,e\} \cup \{d,e,f\}$$

$$= \{a,b,c,d,e,d,e,f\}$$

$$\text{Answer: } \{a,b,c,d,e,f\}$$

Find $(B \cup C)'$

I have to work on the inside of the parenthesis first.

So I will first find:

$$B \cup C$$

$$B \cup C = \{c,d,e\} \cup \{d,e,f\}$$

$$B \cup C = \{c,d,e,d,e,f\}$$

$$B \cup C = \{c,d,e,f\}$$

Now I can do the complement.

I can replace the inside of the parenthesis with $\{c,d,e,f\}$ and proceed to find its complement.

$$(B \cup C)' = (c,d,e,f)' \text{ (my answer will be all the elements of set } U \text{ that are not in this set.)}$$

$$\text{Answer: } \{a,b\}$$

Find $A \cup (B \cup C)'$

First I need to simplify the parenthesis $(B \cup C)'$

I just figured out that $(B \cup C)' = \{a,b\}$, so I will use the work I have already done

$$A \cup (B \cup C)'$$

$$= A \cup \{a, b\}$$

$$= \{a,b,c\} \cup \{a,b\}$$

$$= \{a,b,c,a,b\}$$

$$\text{Answer: } \{a,b,c\}$$

Find $A' \cap (B \cap C')$

I need to simplify the inside of the parenthesis first.

$$(B \cap C')$$

$$= \{c, d, e\} \cap \{a, b, c\}$$

$$= \{c\}$$

$$A' \cap (B \cap C')$$

$$= A' \cap \{c\}$$

$$= \{d, e, f\} \cap \{c\}$$

Answer: $\emptyset$ (empty set)

#21-32: Find the following sets.

$$U = \{1, 2, 3, 4, 5, 6\} \quad A = \{1, 2, 3\} \quad B = \{2, 3, 4\} \quad C = \{1, 5\}$$

$$21) A \cap C$$

$$22) B \cap C$$

$$23) A \cup C$$

$$24) B \cup C$$

$$25) A \cap B \cup C$$

$$26) A \cup B \cap C$$

$$27) B \cup C \cap A$$

$$28) B \cap A \cup C$$

$$29) A' \cap B$$

$$30) A \cap B'$$

$$31) A' \cup B \cap C'$$

$$32) B' \cap A \cup C'$$

#33-44: Find the following sets.

$$U = \{a, b, c, d\} \quad A = \{a, b, c\} \quad B = \{b, c, d\} \quad C = \{a, d\}$$

$$33) A \cap C'$$

$$34) B' \cap C$$

$$35) A' \cup C'$$

$$36) B' \cup C'$$

$$37) A' \cap B \cup C'$$

$$38) A' \cup B' \cap C$$

$$39) B' \cup C' \cap A$$

$$40) B' \cap A' \cup C$$

$$41) A' \cap B'$$

$$42) A \cap B'$$

$$43) A' \cup B' \cap C'$$

$$44) B \cap A' \cup C'$$

#45 – 56: Find the following sets.

$$U = \{1,2,3,4,5,6\} \quad S = \{2,4,6\} \quad T = \{1,2,4\} \quad V = \{4,5,6\}$$

45) $S \cup (T \cap V)$

46) $(S \cup T) \cap V$

47) $(S \cup T)'$

48) $(V \cup S)'$

49) $S \cap (V \cap T')$

50) $(S' \cap V') \cup T$

51) $(S' \cup V') \cap T$

52) $S' \cup T \cap V'$

53) $T \cup V' \cup S'$

54) $T \cup V' \cap S'$

55) $(V \cap T)' \cup S$

56) $V \cup (S \cap T)'$